Section Check In – 1.06 Exponentials and Logarithms

Questions

1. Solve the equation $4^x = 25$.

2. Evaluate $3\log_6 2 + \log_6 9 - \log_6 2$.

3. Sketch the graph of $y = \left|e^{3x} - 6\right|$. Indicate clearly any points where the curve touches the axes and any asymptotes.

4. Find the exact solution to the equation $2\ln x + \ln(4) = 8$.

5. Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 1$. Give your answer in the form $y = ax + b$ where a and b are exact values.

6. (i) Sketch the graph of $f(x) = 2 + \ln(3 + x)$. Indicate clearly any points where the curve crosses the axes and any asymptotes.
   (ii) Find $f^{-1}(x)$ and find the largest possible domain and range for this function.
   (iii) Sketch $f(x)$ and $f^{-1}(x)$ on the same axis showing the relationship between them.

7. Solve the equation $3^{2x} - 6 \times 3^x + 8 = 0$.

8. Find the coordinates of the stationary point on the curve $y = 2xe^{(x+4)}$.

9. The average number of visitors per month at a local museum between the year 2000 and 2005 is shown below.

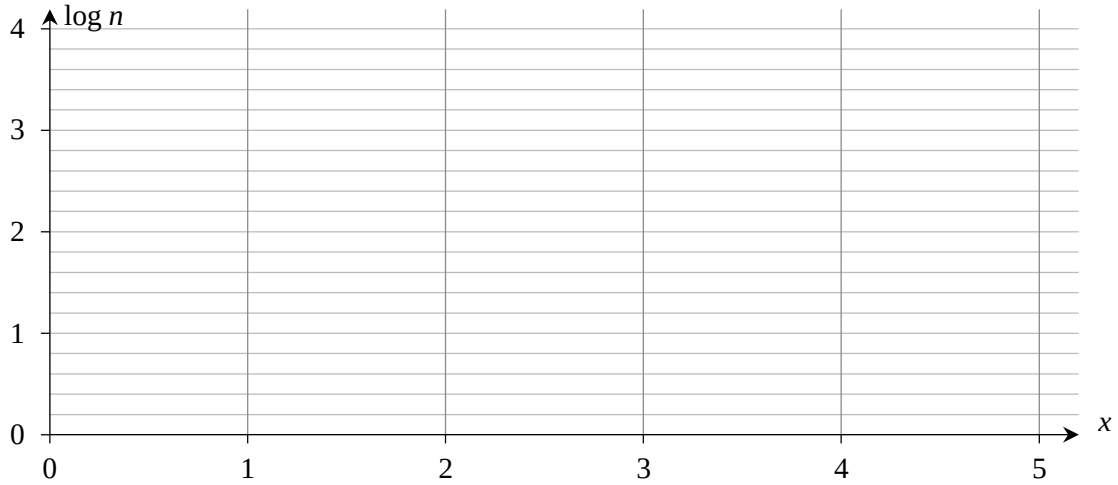
Year	2000	2001	2002	2003	2004	2005
Average number of visitors per month n	350	455	585	761	989	1285

The average number of visitors per month can be modelled by an equation of the form $n = kb^x$, where x is the number of years after 2000.

(i) Using base 10, show that this equation can be written as $\log n = \log k + x \log b$.

(ii) Fill in the table below and draw a graph to plot $\log n$ against x.

x	0	1	2	3	4	5
$\log n$						



(iii) Draw a line of best fit on your graph and use it to estimate the values of k and b.

(iv) Use your values of k and b to predict the average number of visitors per month in the year 2010. Comment on the reliability of this estimate.

10. The radioactive decay of a substance is given by $R = 500e^{-kt}$, $t \geq 0$, where R is the number of atoms after time, t, years and k is a positive constant.

(i) How many atoms were there when the substance started to decay?

The half-life of the substance is 730 years.

(ii) Find the value of k to three decimal places.

(iii) How many atoms will be left after 1000 years?

(iv) Find the rate at which the substance is decaying when $t = 20$.

(v) Sketch the graph of R against t.

Worked solutions

1. $4^x = 25$

$\log 4^x = \log 25$

$x \log 4 = \log 25$

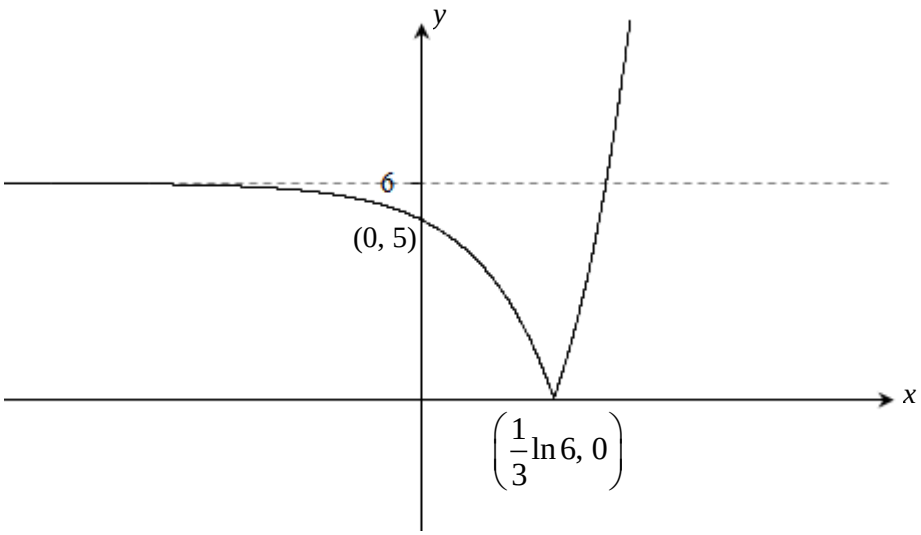
$x = \dfrac{\log 25}{\log 4} = 2.32$ (3 s.f.)

2. $3\log_6 2 + \log_6 9 - \log_6 2$

$= \log_6 2^3 + \log_6 9 - \log_6 2$

$= \log_6\left(\dfrac{8 \times 9}{2}\right) = \log_6 36 = 2$

3.



4. $2\ln x + \ln(4) = 8$

$\ln x^2 + \ln(4) = 8$

$\ln(4x^2) = 8$

$4x^2 = e^8$

$x^2 = \dfrac{e^8}{4}$

$x = \sqrt{\dfrac{e^8}{4}}$ Positive answer only.

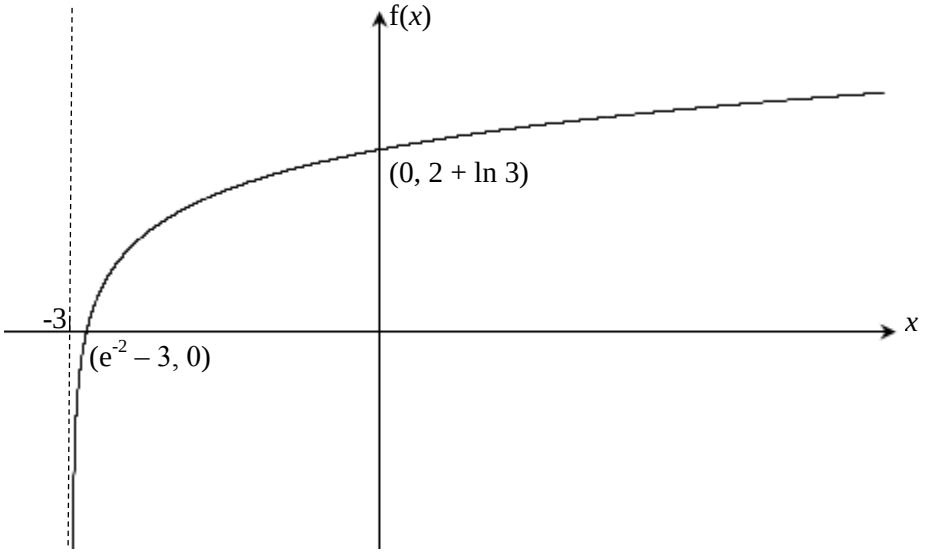
5. $y = e^{2x}$ $y = e^2$ when $x = 1$

$\dfrac{dy}{dx} = 2e^{2x} = 2e^2$ when $x = 1$

$y - e^2 = 2e^2(x - 1) = 2e^2 x - 2e^2$

$y = 2e^2 x - e^2$

6. (i)



(ii) $y = 2 + \ln(3 + x)$

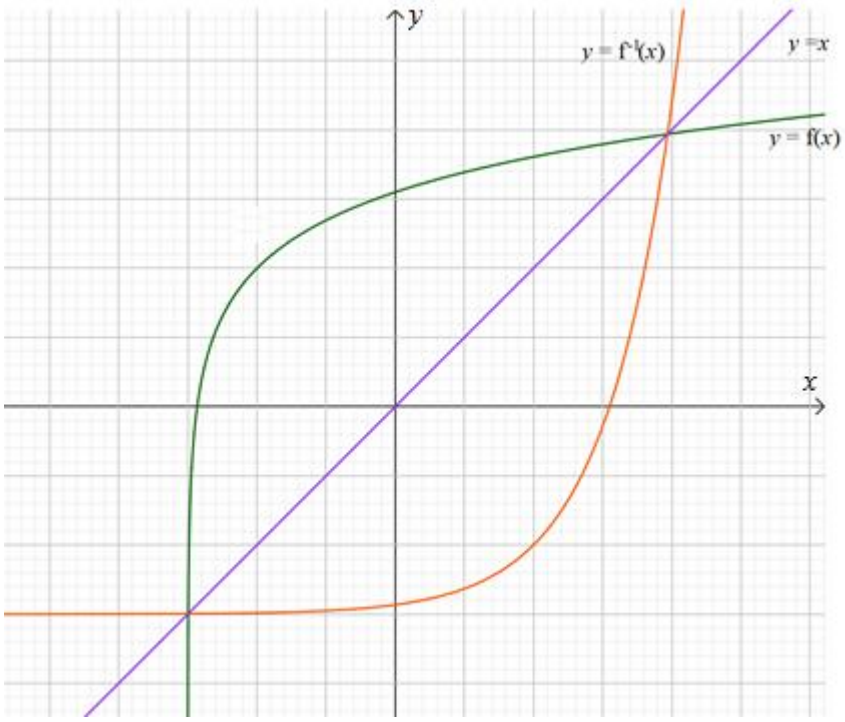
$y - 2 = \ln(3 + x)$

$e^{(y-2)} = 3 + x$

$x = e^{(y-2)} - 3$

So $f^{-1}(x) = e^{(x-2)} - 3$. Domain is all real numbers. Range is $f^{-1}(x) > -3$.

(iii)



$f^{-1}(x)$ is a reflection of $f(x)$ in the line $y = x$.

7. $3^{2x}-6\times3^{x}+8=0$ Let $y=3^{x}$

$y^{2}-6y+8=0$

$(y-2)(y-4)=0 \quad y=2$ or $y=4$

If $y=2$ then $3^{x}=2$ so $x=\dfrac{\log 2}{\log 3}=0.631$ (3 d.p.)

If $y=4$ then $3^{x}=4$ so $x=\dfrac{\log 4}{\log 3}=1.262$ (3 d.p.)

8. $y=2xe^{(x+4)}=uv$ where $u=2x$ and $v=e^{(x+4)}$

$\dfrac{du}{dx}=2,\ \dfrac{dv}{dx}=e^{(x+4)}$

$\dfrac{dy}{dx}=2e^{(x+4)}+2xe^{(x+4)}=2e^{(x+4)}(x+1)$

$\dfrac{dy}{dx}=0$ when $x=-1$ so stationary point is $(-1,-2e^{3})$

9. (i) $n=kb^{x}$

$\log_{10} n=\log_{10} kb^{x}$

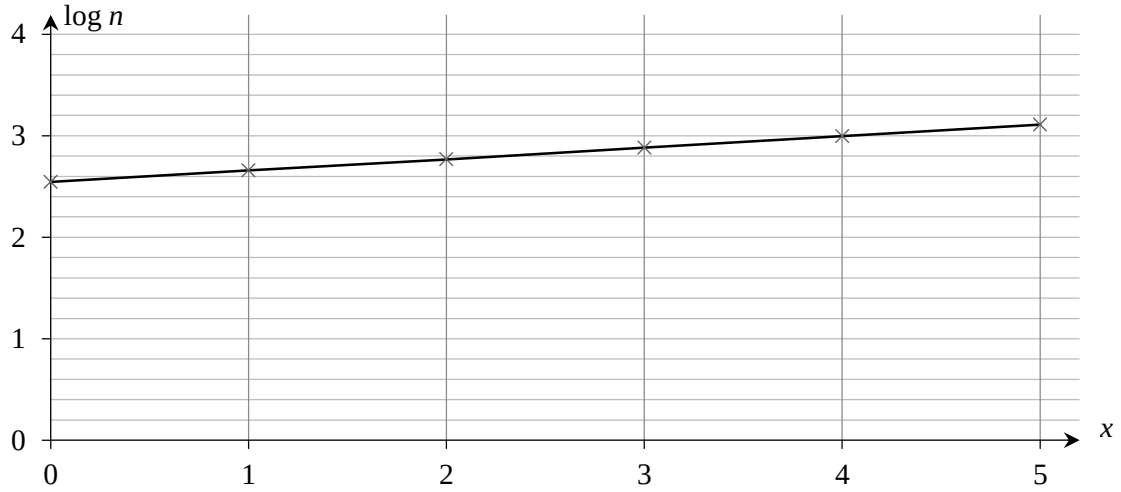
$\log_{10} n=\log_{10} k+\log_{10} b^{x}$

$\log_{10} n=\log_{10} k+x\log_{10} b$

(ii)

x	0	1	2	3	4	5
$\log n$	2.54	2.66	2.77	2.88	3.00	3.11

(ii) and (iii)



Intercept is $2.54=\log_{10} k$ so $k=350$

Gradient is $\dfrac{0.57}{5}=0.114=\log_{10} b$ so $b=1.3$

(iv) In 2010, $x = 10$ and $n = 350 \times 1.3^{10} = 4825$.
This is extrapolation. 2010 is 5 years after the data given. The museum may not have the capacity for 4825 visitors per month or it may close or change etc. The figure is not reliable.

10. (i) 500

(ii) $250 = 500e^{-730k}$

$$\frac{1}{2} = e^{-730k}$$

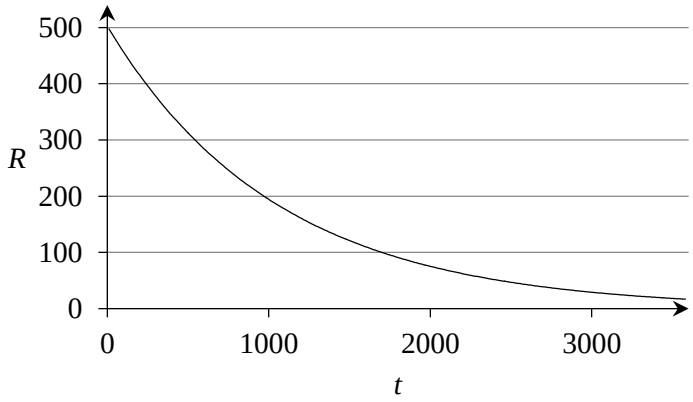
$$\ln\frac{1}{2} = -730k$$

$$k = \frac{-\ln 0.5}{730} = 0.00095$$

(iii) When $t = 1000$ $R = 500e^{-0.95} = 193$ atoms.

(iv) $\frac{dR}{dt} = 500 \times -0.00095e^{-0.00095t}$
When $t = 20$, rate of decay is -0.466 atoms per year.

(v)



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