# Section Check In – 1.07 Differentiation

## Questions

1. Given that $\mathrm{f}(x) = \frac{20}{x}$, find the value of $\mathrm{f}''(2)$.

2. Find the equation of the tangent to the curve $y = x^3 - 6x - 1$ at the point $(-2, 3)$.

3. Given that $\mathrm{f}(x) = 2x^3$, use differentiation from first principles to show that $\mathrm{f}'(x) = 6x^2$.

4. The equation of a curve is $y = 2x + \frac{18}{x} + 5$. Find the coordinates of the stationary points and determine whether each stationary point is a minimum or a maximum.

5. A balloon is released into the air at a funfair. When the balloon is a horizontal distance of $x$ km from the funfair, its height is $H$ m where $H = 56\sqrt{x} - 8x$. Find the greatest height reached by the balloon.

 

## Worked solutions

1. Writing $\mathrm{f}(x) = 20x^{-1}$ and differentiating twice, $\mathrm{f}'(x) = -20x^{-2}$ and $\mathrm{f}''(x) = 40x^{-3}$

   Substituting $x = 2$ gives $\mathrm{f}''(2) = 40 \times 2^{-3} = 40 \times \frac{1}{8} = 5$

2. Differentiating $y = x^3 - 6x - 1$ gives $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 6$

   When $x = -2$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3 \times 4 - 6 = 6$

   Equation of tangent is $y - 3 = 6(x + 2)$ or $y = 6x + 15$

3. $$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h} = \lim_{h \to 0} \frac{2(x+h)^3 - 2x^3}{h} = \lim_{h \to 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - 2x^3}{h}$$

   $$= \lim_{h \to 0} \frac{6x^2h + 6xh^2 + 2h^3}{h} = \lim_{h \to 0} (6x^2 + 6xh + 2h^2)$$, dividing by $h$

   As $h$ tends to zero, $6xh \to 0$ and $2h^2 \to 0$ leaving $\mathrm{f}'(x) = 6x^2$

4. From $y = 2x + 18x^{-1} + 5$, differentiation gives $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2 - 18x^{-2}$

   For stationary point, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ giving $x^{-2} = \frac{2}{18}$ and therefore $x^2 = 9$ and $x = \pm 3$

   To consider nature of stationary point, use second derivative $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 36x^{-3}$

   When $x = -3$, $y = -6 - 6 + 5 = -7$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{36}{(-3)^3} = -\dfrac{36}{27}$; hence $(-3, -7)$ is maximum

   When $x = 3$, $y = 6 + 6 + 5 = 17$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{36}{3^3} = \dfrac{36}{27}$; hence $(3, 17)$ is minimum

5. Greatest height corresponds to maximum value of $H$, i.e. when $\dfrac{\mathrm{d}H}{\mathrm{d}x} = 0$

   $H = 56x^{\frac{1}{2}} - 8x$ and differentiating gives $\dfrac{\mathrm{d}H}{\mathrm{d}x} = 28x^{-\frac{1}{2}} - 8$

   For stationary point, $28x^{-\frac{1}{2}} = 8$ leading to $x = \frac{49}{4}$

   Since $\dfrac{\mathrm{d}^2H}{\mathrm{d}x^2} = -14x^{-\frac{3}{2}}$ is negative, stationary point is a maximum

   When $x = \frac{49}{4}$, $H = 56 \times \sqrt{\frac{49}{4}} - 8 \times \frac{49}{4} = 98$

   Greatest height reached is $98$ metres

 

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