

Section Check In – 1.07 Differentiation

Questions

1. Given that $f(x) = \frac{20}{x}$, find the value of $f''(2)$.
2. Find the equation of the tangent to the curve $y = x^3 - 6x - 1$ at the point $(-2, 3)$.
3. Given that $f(x) = 2x^3$, use differentiation from first principles to show that $f'(x) = 6x^2$.
4. The equation of a curve is $y = 2x + \frac{18}{x} + 5$. Find the coordinates of the stationary points and determine whether each stationary point is a minimum or a maximum.
5. A balloon is released into the air at a funfair. When the balloon is a horizontal distance of x km from the funfair, its height is H m where $H = 56\sqrt{x} - 8x$. Find the greatest height reached by the balloon.

AS LEVEL and STAGE 1 CONTENT
MATHEMATICS A
Section Check In

Worked solutions

1. Writing $f(x) = 20x^{-1}$ and differentiating twice, $f'(x) = -20x^{-2}$ and $f''(x) = 40x^{-3}$
Substituting $x = 2$ gives $f''(2) = 40 \times 2^{-3} = 40 \times \frac{1}{8} = 5$

2. Differentiating $y = x^3 - 6x - 1$ gives $\frac{dy}{dx} = 3x^2 - 6$

When $x = -2$, $\frac{dy}{dx} = 3 \times 4 - 6 = 6$

Equation of tangent is $y - 3 = 6(x + 2)$ or $y = 6x + 15$

3.
$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{2(x+h)^3 - 2x^3}{h} = \lim_{h \rightarrow 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - 2x^3}{h}$$
$$= \lim_{h \rightarrow 0} \frac{6x^2h + 6xh^2 + 2h^3}{h} = \lim_{h \rightarrow 0} (6x^2 + 6xh + 2h^2), \text{ dividing by } h$$

As h tends to zero, $6xh \rightarrow 0$ and $2h^2 \rightarrow 0$ leaving $f'(x) = 6x^2$

4. From $y = 2x + 18x^{-1} + 5$, differentiation gives $\frac{dy}{dx} = 2 - 18x^{-2}$

For stationary point, $\frac{dy}{dx} = 0$ giving $x^{-2} = \frac{2}{18}$ and therefore $x^2 = 9$ and $x = \pm 3$

To consider nature of stationary point, use second derivative $\frac{d^2y}{dx^2} = 36x^{-3}$

When $x = -3$, $y = -6 - 6 + 5 = -7$ and $\frac{d^2y}{dx^2} = \frac{36}{(-3)^3} = -\frac{36}{27}$; hence $(-3, -7)$ is maximum

When $x = 3$, $y = 6 + 6 + 5 = 17$ and $\frac{d^2y}{dx^2} = \frac{36}{3^3} = \frac{36}{27}$; hence $(3, 17)$ is minimum

5. Greatest height corresponds to maximum value of H , i.e. when $\frac{dH}{dx} = 0$

$H = 56x^{\frac{1}{2}} - 8x$ and differentiating gives $\frac{dH}{dx} = 28x^{-\frac{1}{2}} - 8$

For stationary point, $28x^{-\frac{1}{2}} = 8$ leading to $x = \frac{49}{4}$

Since $\frac{d^2H}{dx^2} = -14x^{-\frac{3}{2}}$ is negative, stationary point is a maximum

When $x = \frac{49}{4}$, $H = 56 \times \sqrt{\frac{49}{4}} - 8 \times \frac{49}{4} = 98$

Greatest height reached is 98 metres

AS LEVEL and STAGE 1 CONTENT
MATHEMATICS A
Section Check In

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