

Section Check In – 1.08 Integration

Questions

1. Find $\int \left(3x + \frac{2}{x}\right)^2 dx$.
2. **In this question you must show detailed reasoning.**
Evaluate $\int_1^4 \left(3\sqrt{x} + \frac{4}{\sqrt{x}}\right) dx$.
3. (i) Given that $y = 3x^{\frac{1}{2}} + 12x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$, show that $\frac{dy}{dx} = \frac{3(x-2)^2}{2x^2\sqrt{x}}$.
(ii) Deduce that $\int_1^9 \frac{(x-2)^2}{2x^2\sqrt{x}} dx = \frac{50}{81}$.
4. **In this question you must show detailed reasoning.**
The equation of a curve is $y = 3x^2 - 18x + 15$. Find the area of the region enclosed by the curve and the x -axis.
5. The mass of a chemical at a time t hours after the start of an experiment is m grams. The rate of increase of the mass is given by $\frac{dm}{dt} = 2 + 0.1t$. One hour after the start of the experiment, the mass was 9.4 grams. Find the mass of the substance at a time ten hours after the start of the experiment.

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Worked solutions

1. Expanding the integrand, $\int (9x^2 + 12 + 4x^{-2}) dx$ giving $3x^3 + 12x - 4x^{-1} + c$

$$2. \int_1^4 (3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}) dx = \left[\frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^4 = \left[2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} \right]_1^4 = 2 \times 8 + 8 \times 2 - (2 \times 1 + 8 \times 1) = 22$$

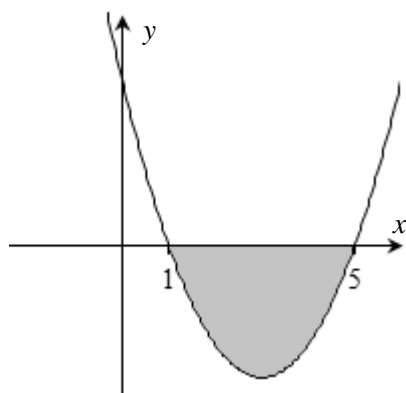
$$3. (i) \frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - 6x^{-\frac{3}{2}} + 6x^{-\frac{5}{2}} = \frac{3}{2\sqrt{x}} - \frac{6}{x\sqrt{x}} + \frac{6}{x^2\sqrt{x}} = \frac{3x^2 - 12x + 12}{2x^2\sqrt{x}} = \frac{3(x^2 - 4x + 4)}{2x^2\sqrt{x}} = \frac{3(x-2)^2}{2x^2\sqrt{x}}$$

(ii) By the Fundamental Theorem of Calculus,

$$\int_1^9 \frac{3(x-2)^2}{2x^2\sqrt{x}} dx = \left[3x^{\frac{1}{2}} + 12x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}} \right]_1^9 = (3 \times 3 + 12 \times \frac{1}{3} - 4 \times \frac{1}{27}) - (3 + 12 - 4) = \frac{50}{27}$$

$$\text{Dividing by 3, } \int_1^9 \frac{(x-2)^2}{2x^2\sqrt{x}} dx = \frac{50}{81}$$

4. $y = 3x^2 - 18x + 15 = 3(x^2 - 6x + 5) = 3(x-1)(x-5)$ so curve meets x -axis at 1 and 5



$$\int_1^5 (3x^2 - 18x + 15) dx = \left[x^3 - 9x^2 + 15x \right]_1^5 = 125 - 225 + 75 - (1 - 9 + 15) = -32$$

Answer is negative because region is below the x -axis; area is 32 square units

5. Integrating, $m = 2t + 0.05t^2 + c$

When $t = 1$, $m = 9.4$ and so $9.4 = 2 + 0.05 + c$ giving $c = 7.35$

Formula for the mass is $m = 2t + 0.05t^2 + 7.35$

When $t = 10$, $m = 2 \times 10 + 0.05 \times 100 + 7.35 = 32.35$

After 10 hours, mass is 32 grams (rounding to take account of possible experimental error)

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