OCR
Oxford Cambridge and RSA

Section Check In – 1.02 Algebra and Functions

Questions

1. Use the factor theorem to show that $(2x-1)$ is a factor of $2x^3-3x^2+7x-3$.

2. Simplify $\sqrt{12}+6\times 3^{-\frac{1}{2}}-27^{\frac{1}{2}}$.

3. (i) Express each of $x^2-6x+27$ and $1+6x-x^2$ in completed square form.
   (ii) Sketch the curves $y=x^2-6x+27$ and $y=1+6x-x^2$ on the same axes, and show that the shortest distance between the two curves is 8.

4. Show that the straight line $y=x+k$ meets the curve $x^2+3xy+y^2=5$ in two distinct points for all values of the constant k.

5. An object is projected vertically upwards from ground level and its height, H metres, after t seconds is given by the formula

$$H=60t-5t^2.$$

   Show that the object is above a height of 90 metres for approximately 8.5 seconds.

Worked solutions

1. Let $\mathrm{f}(x) = 2x^3 - 3x^2 + 7x - 3$

   $\mathrm{f}\left(\frac{1}{2}\right) = 2\times\frac{1}{8} - 3\times\frac{1}{4} + 7\times\frac{1}{2} - 3 = \frac{1}{4} - \frac{3}{4} + 3\frac{1}{2} - 3 = 0$

   Since $\mathrm{f}\left(\frac{1}{2}\right) = 0$, by factor theorem $x = \frac{1}{2}$ or $2x-1$ is a factor

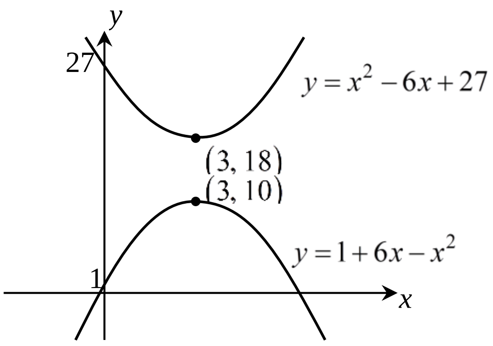
2. Simplifying, $\sqrt{12} + 6\times 3^{-\frac{1}{2}} - 27^{\frac{1}{2}} = \sqrt{4\times 3} + \dfrac{6}{\sqrt{3}} - \sqrt{9\times 3} = 2\sqrt{3} + \dfrac{6\sqrt{3}}{3} - 3\sqrt{3} = \sqrt{3}$

3. (i) $x^2 - 6x + 27 = (x-3)^2 - 9 + 27 = (x-3)^2 + 18$

   $1 + 6x - x^2 = 1 - (x-3)^2 + 9 = 10 - (x-3)^2$

   (ii) $y = (x-3)^2 + 18$ has minimum at $(3, 18)$

   $y = 10 - (x-3)^2$ has maximum at $(3, 10)$



   Shortest distance between curves is the distance between their stationary points.
   Shortest distance is $18 - 10 = 8$

4. Substituting $y = x + k$ in the equation of the curve, $x^2 + 3x(x+k) + (x+k)^2 = 5$

   Expanding, $x^2 + 3x^2 + 3kx + x^2 + 2kx + k^2 - 5 = 0$

   Simplifying, $5x^2 + 5kx + k^2 - 5 = 0$

   Discriminant, $b^2 - 4ac = (5k)^2 - 4\times 5\times(k^2 - 5) = 25k^2 - 20k^2 + 100 = 5k^2 + 100$

   For all values of k, $k^2 \geq 0$ and therefore $5k^2 + 100 > 0$

   Since the discriminant is positive, the quadratic equation has two distinct roots and so the line meets the curve in two distinct points.

5. Times when object is at a height of 90 metres found by substituting $H = 90$

   Equation is $90 = 60t - 5t^2$ which simplifies to $t^2 - 12t + 18 = 0$

   Using quadratic formula, $t = 1.757...$ or $t = 10.242...$

   Object at height of 90 metres after $1.757...$ seconds (going up) and after $10.242...$ seconds (going down)

   Object above this height for $10.242... - 1.757... = 8.485...$ seconds, i.e. for 8.49 seconds (to 3 significant figures)

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