

## Section Check In – 1.02 Algebra and Functions

### Questions

1. Use the factor theorem to show that  $(2x - 1)$  is a factor of  $2x^3 - 3x^2 + 7x - 3$ .
2. Simplify  $\sqrt{12} + 6 \times 3^{-\frac{1}{2}} - 27^{\frac{1}{2}}$ .
3. (i) Express each of  $x^2 - 6x + 27$  and  $1 + 6x - x^2$  in completed square form.  
(ii) Sketch the curves  $y = x^2 - 6x + 27$  and  $y = 1 + 6x - x^2$  on the same axes, and show that the shortest distance between the two curves is 8.
4. Show that the straight line  $y = x + k$  meets the curve  $x^2 + 3xy + y^2 = 5$  in two distinct points for all values of the constant  $k$ .
5. An object is projected vertically upwards from ground level and its height,  $H$  metres, after  $t$  seconds is given by the formula

$$H = 60t - 5t^2.$$

Show that the object is above a height of 90 metres for approximately 8.5 seconds.

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### Worked solutions

1. Let  $f(x) = 2x^3 - 3x^2 + 7x - 3$

$$f\left(\frac{1}{2}\right) = 2 \times \frac{1}{8} - 3 \times \frac{1}{4} + 7 \times \frac{1}{2} - 3 = \frac{1}{4} - \frac{3}{4} + 3\frac{1}{2} - 3 = 0$$

Since  $f\left(\frac{1}{2}\right) = 0$ , by factor theorem  $x = \frac{1}{2}$  or  $2x - 1$  is a factor

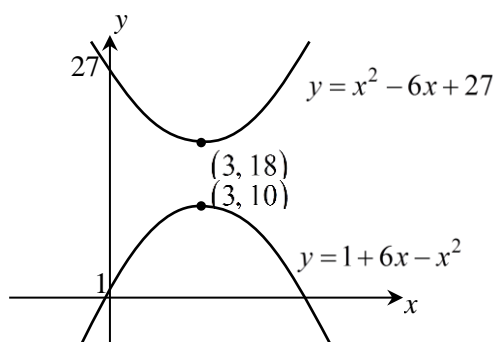
2. Simplifying,  $\sqrt{12} + 6 \times 3^{-\frac{1}{2}} - 27^{\frac{1}{2}} = \sqrt{4 \times 3} + \frac{6}{\sqrt{3}} - \sqrt{9 \times 3} = 2\sqrt{3} + \frac{6\sqrt{3}}{3} - 3\sqrt{3} = \sqrt{3}$

3. (i)  $x^2 - 6x + 27 = (x - 3)^2 - 9 + 27 = (x - 3)^2 + 18$

$$1 + 6x - x^2 = 1 - (x - 3)^2 + 9 = 10 - (x - 3)^2$$

(ii)  $y = (x - 3)^2 + 18$  has minimum at  $(3, 18)$

$$y = 10 - (x - 3)^2 \text{ has maximum at } (3, 10)$$



Shortest distance between curves is the distance between their stationary points.

Shortest distance is  $18 - 10 = 8$

4. Substituting  $y = x + k$  in the equation of the curve,  $x^2 + 3x(x + k) + (x + k)^2 = 5$

Expanding,  $x^2 + 3x^2 + 3kx + x^2 + 2kx + k^2 - 5 = 0$

Simplifying,  $5x^2 + 5kx + k^2 - 5 = 0$

Discriminant,  $b^2 - 4ac = (5k)^2 - 4 \times 5 \times (k^2 - 5) = 25k^2 - 20k^2 + 100 = 5k^2 + 100$

For all values of  $k$ ,  $k^2 \geq 0$  and therefore  $5k^2 + 100 > 0$

Since the discriminant is positive, the quadratic equation has two distinct roots and so the line meets the curve in two distinct points.

5. Times when object is at a height of 90 metres found by substituting  $H = 90$

Equation is  $90 = 60t - 5t^2$  which simplifies to  $t^2 - 12t + 18 = 0$

Using quadratic formula,  $t = 1.757\dots$  or  $t = 10.242\dots$

Object at height of 90 metres after 1.757... seconds (going up) and after 10.242... seconds (going down)

Object above this height for  $10.242\dots - 1.757\dots = 8.485\dots$  seconds, i.e. for 8.49 seconds (to 3 significant figures)

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