Section Check In – 1.03 Coordinate Geometry

Questions

1. Find the equation of the straight line through the points $(-1,\ 4)$ and $(3,\ -2)$. Give your answer in the form $ax+by+c=0$.

2. Find the centre and radius of the circle with equation $x^2+y^2+2x-6y-15=0$.

3. The coordinates of three points are $A(-2, 5)$, $B(4, 7)$ and $C(6, 1)$.
   (i) Show that AB is perpendicular to BC.
   (ii) Explain why the mid-point of AC is the centre of the circle through A, B and C.
   (iii) Find the equation of the circle through A, B and C.

4. The point P has coordinates (a, b) and the point Q has coordinates $(8, 7)$. The gradient of PQ is $\frac{1}{2}$ and the distance PQ is $3\sqrt{5}$. Find the possible values of a and b.

5. A circle with centre at the origin has radius $2\sqrt{5}$. A straight line has equation $y=2x+k$ and meets the circle at two distinct points. Find the possible values of k.

6. Alan is conducting an experiment involving the variables x and T. The measurements are given in the following table.

x	5	6	7	8	9	10	11
T	65	80	101	120	125	167	188

Alan suspects that the variables are linked by an equation of the form $T=kx^{\frac{3}{2}}+c$. By plotting values of T against values of $x^{\frac{3}{2}}$, comment fully on Alan's suspicions and, if appropriate, suggest values for the constants k and c.

Worked solutions

1. Gradient of line is $\frac{-2-4}{3--1} = -\frac{6}{4} = -\frac{3}{2}$

   Equation of line is $y-4 = -\frac{3}{2}(x--1)$ or $y-4 = -\frac{3}{2}x - \frac{3}{2}$

   Multiplying both sides by 2 and rearranging to required form, equation is $3x+2y-5=0$

2. Equation is $x^2+2x+y^2-6y-15=0$

   Completing the square twice, $(x+1)^2-1+(y-3)^2-9-15=0$

   Simplifying, equation is $(x+1)^2+(y-3)^2=25$

   Hence centre is $(-1, 3)$ and radius is 5

3. (i) Gradient of AB is $\frac{7-5}{4--2} = \frac{2}{6} = \frac{1}{3}$; gradient of BC is $\frac{1-7}{6-4} = \frac{-6}{2} = -3$

   Product of gradients $= \frac{1}{3} \times (-3) = -1$ and so lines are perpendicular

   (ii) Since angle $ABC = 90^\circ$, the angle in a semicircle property means that AC is diameter of circle through B. Mid-point of the diameter AC is the centre of the circle

   (iii) Mid-point M of AC is $\left(\frac{-2+6}{2}, \frac{5+1}{2}\right) = (2, 3)$

   Radius $MA = \sqrt{(-2-2)^2+(5-3)^2} = \sqrt{20}$

   Equation of circle is $(x-2)^2+(y-3)^2=20$

4. Gradient of $PQ = \frac{1}{2}$, giving the equation $\frac{7-b}{8-a} = \frac{1}{2}$ and so $2(7-b) = 8-a$

   Simplifying, $a-2b=-6$

   Distance $PQ = 3\sqrt{5}$, giving the equation $(8-a)^2+(7-b)^2 = 9\times 5$

   Expanding and simplifying, $a^2-16a+b^2-14b+68=0$

   From first equation, $a=2b-6$

   Substituting in second equation, $(2b-6)^2-16(2b-6)+b^2-14b+68=0$

   Simplifying, $b^2-14b+40=0$ and, factorising, $(b-10)(b-4)=0$

   Solving, $b=10$ or $b=4$, giving $a=14$ or $a=2$

   Possible values are $a=14, b=10$ and $a=2, b=4$

5. Equation of circle is $x^2+y^2=20$

   Line meets circle where $x^2+(2x+k)^2=20$

   Expanding and simplifying, $5x^2+4kx+k^2-20=0$

   For two distinct roots, discriminant $b^2-4ac>0$, giving $(4k)^2-4\times 5\times (k^2-20)>0$

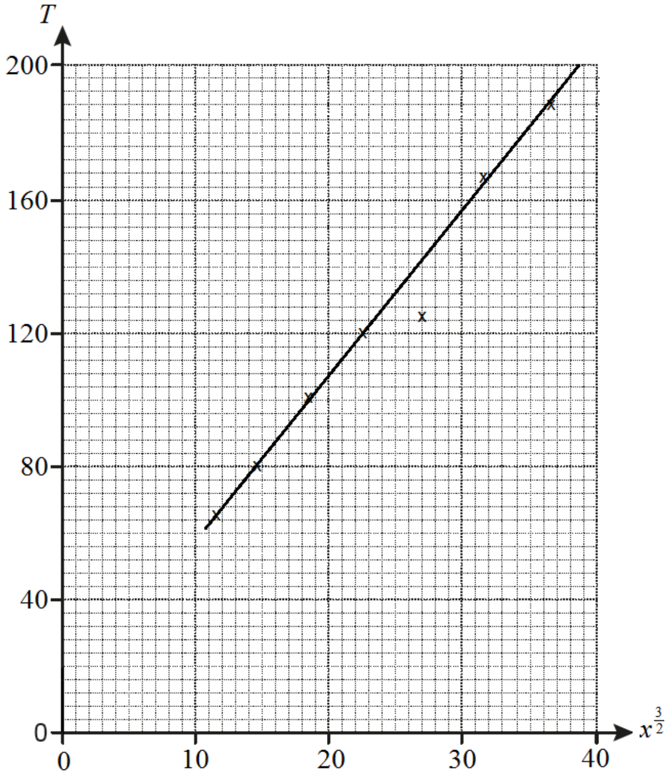
   Simplifying, $k^2-100<0$ and so $(k+10)(k-10)<0$

   Possible values of k are those for which $-10<k<10$

6. Forming table showing values of $x^{\frac{3}{2}}$ and T,

$x^{\frac{3}{2}}$	11.2	14.7	18.5	22.6	27.0	31.6	36.5
T	65	80	101	120	125	167	188

If Alan is correct, plotting these points should show the points lying on a straight line (more or less, allowing for experimental inaccuracies)



Checking the graph, the points (except for one) do lie close to a straight line
Ignoring the point $(27.0, 125)$ with the assumption that there was an error made with this reading
For $T = kx^{\frac{3}{2}} + c$, value of k is gradient of line in graph
Line through $(10, 60)$ and $(38.5, 200)$ has gradient $\frac{200-60}{38.5-10} = 4.912...$
Substituting values gives $c = 10.877...$
Ignoring the one apparently wrong reading, Alan's suspicions are justified and T is given approximately by $T = 4.9x^{\frac{3}{2}} + 11$, rounding the two constants to 2 significant figures

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