

## Section Check In – 1.01 Proof

### Questions

1. Prove by exhaustion that, in the set of natural numbers less than 50, there are fewer square numbers than prime numbers.
2. Let  $p$  be a prime number such that  $2 < p < 50$ . Prove, by exhaustion, that for all such  $p$ ,  $(p-1)(p+1)$  is divisible by 8.
3. The following result is known as the “difference of two squares”  
$$(a^2 - b^2) = (a+b)(a-b).$$
Find and prove a similar formula for the difference of two cubes  $(a^3 - b^3)$ .
4. Prove that  $n - 3$  is even if and only if  $n$  is odd.
5. A new hotel is to be built and will have a cylindrical swimming pool. Given that the pool’s tiled surface area will be  $225 \text{ m}^2$  show that its maximum volume is approximately  $366 \text{ m}^3$ .

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## Worked solutions

1. Proof by exhaustion:

All square numbers less than 50 are 1, 4, 9, 16, 25, 36, 49

All prime numbers less than 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

Therefore there are fewer squares than primes.

2. Proof by exhaustion:

$$2(4) = 8$$

$$4(6) = 24 = 3(8)$$

$$6(8)$$

$$10(12) = 15(8)$$

$$12(14) = 21(8)$$

$$16(18) = 36(8)$$

$$18(20) = 45(8)$$

$$22(24) = 66(8)$$

$$28(30) = 105(8)$$

$$30(32) = 120(8)$$

$$36(38) = 171(8)$$

$$40(42) = 210(8)$$

$$42(44) = 231(8)$$

$$46(48) = 276(8)$$

Therefore the product of the two numbers adjacent, at either side, of the low primes ( $< 50$ ) is divisible by 8.

3.  $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

Proof:

$$\text{RHS} = (a - b)(a^2 + ab + b^2)$$

$$= a^3 + a^2b + ab^2 - ba^2 - ab^2 - b^3$$

$$= a^3 - b^3$$

$$= \text{LHS}$$

Hence proven.

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4. Need  $\Rightarrow$  and  $\Leftarrow$  for if and only if proofs.  
Therefore, initially need to prove that  $n - 3$  is even  $\Rightarrow n$  is odd.

Now  $n - 3$  is even so  $n - 3 = 2a$  for  $a \in \mathbb{Z}$ .  
So  $n = 2a + 3 = 2a + 2 + 1 = 2(a + 1) + 1$ .  
 $2(a + 1)$  is divisible by 2 and is therefore even.  
Hence  $2(a + 1) + 1 = n$  is therefore odd.

Now need to prove that  $n$  odd  $\Rightarrow n - 3$  is even.

If  $n$  odd then  $n = b + 1$  for  $b$  even.  
So  $n - 3 = b + 1 - 3 = b - 2$ .  
Now  $b$  is even  $\therefore b$  is divisible by 2 and  $-2 = 2(-1)$  is also divisible by 2.  
So  $n - 3$  is divisible by 2 and is therefore even.

Therefore  $n - 3$  is even if and only if  $n$  is odd.

5. Volume is  $\pi r^2 h$   
Surface area is  $\pi r^2 + 2\pi r h = 225$

$$\text{So } h = \frac{225 - \pi r^2}{2\pi r} \text{ and } V = \pi r^2 \left( \frac{225 - \pi r^2}{2\pi r} \right) = \frac{225}{2} r - \frac{\pi}{2} r^3$$

For the maximum volume we need  $\frac{dV}{dr} = 0$ , i.e.  $\frac{dV}{dr} = \frac{225}{2} - \frac{3\pi}{2} r^2 = 0$

$$\text{So } r = \sqrt{\frac{225}{3\pi}}$$

Substituting this into our equations for  $h$  and  $V$  gives  $V = 366.45... \text{ m}^3$ .

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